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ON THE PERFORMANCE CALCULATIONS OF THREE PHASE TRANSMISSION LINES BY RELAXATION METHOD

R. N. BASU* AND S. N. DUTTA

DEPARTMENT OF APPLIED PHYSICS, CALCUTTA UNIVERSITY

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ABSTRACT. The application of relaxation technique in the nominal-T method of solution of transmission line problem is discussed with an illustration. The principle underlying the relaxational solution of a.c. networks having complex circuit constants (i.e., considering the reactances along with the resistances in different branches is utilised and the results thus obtained are compared with those calculated by normal method of solution. It also discusses about the application of the method to the solution of long transmission lines where the equivalent-T method is used instead of the nominal-T method normally used for medium lines.

INTRODUCTION

The performance calculations of three phase transmission lines under different given conditions at the receiving end and sending end are quite familiar in Electrical Engineering. Of the different methods available for such calculations, the nominal-T method is one. In the nominal-T method the line capacitance is assumed to be localised at the mid-point of the line as shown in Fig. 1. Depending

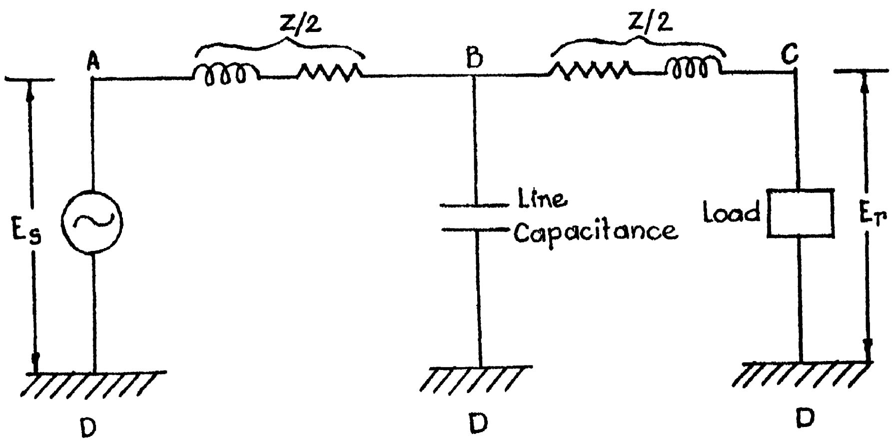

Fig. 1
Diagram for nominal T-method.

*Present address: Research and Development Department, Northern Electric Co., Ottawa, Canada.

on the available informations regarding the sending end and receiving end conditions, three distinct cases can be considered separately, viz.,

(i) when receiving end voltage and load are known,
(ii) when sending end voltage and receiving end impedance are known, and
(iii) when sending end voltage and receiving end load are known.

When the receiving end terminals of the T-network, shown in Fig. 1, are terminated by the load-impedance, the same network can be represented in a convenient form as shown in Fig. 2a. The network thus formed is solved here with the help of relaxation method, and it is shown that such a relaxational solution yields a number of useful informations simultaneously.

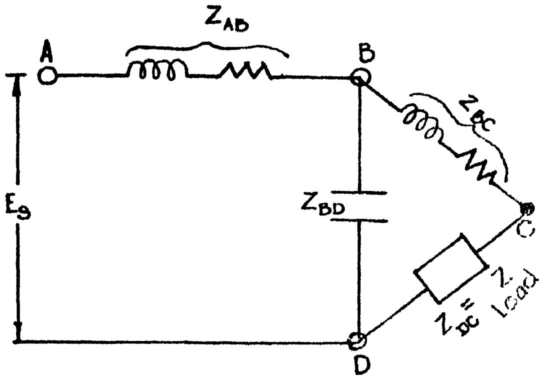

Fig. 2a
Equivalent impedance network diagram nominal T-method.

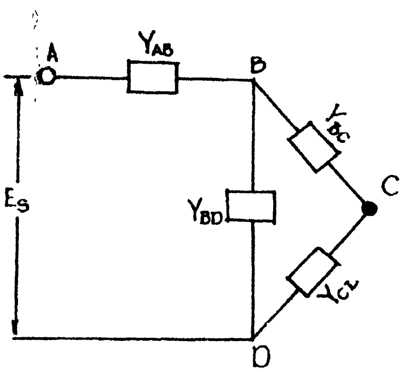

Fig. 2b
Equivalent admittance network diagram for nominal T-method.

In using the technique of relaxation to the solution of a.c. networks, additional complications (in comparison with d.c. networks) arise due to the complex nature of the branch-impedances, which include resistances as well as reactances. Southwell and Black (1938) have applied relaxation method to the solution of an a.c. network having complex circuit constants. The principle underlying such a solution reveals the usefulness of the application of relaxation technique for solving the network shown in Fig. 2a, i.e., for the performance calculations of transmission lines.

PRINCIPLE OF THE METHOD

In Fig. 2b, Y_{AB}, Y_{BC}, ... etc. are equal to the reciprocals of the impedances Z_{AB}, Z_{BC}, ... etc. respectively, and the voltage between A and D stands for the sending end voltage measured between phase and neutral. Now writing the admittance of any branch, say BC, as—

$$Y_{BC} = g_{BC}+jb_{BC} \qquad \ldots \quad (1)$$

and assuming the potentials at the points B and C to be

$$\left.\begin{aligned} v_B &= v_{x(B)}+jv_{y(B)} \\ \text{and} \quad v_C &= v_{x(C)}+jv_{y(C)} \end{aligned}\right\} \qquad \ldots \quad (2)$$

the current flowing from B to C through the branch BC may be represented by

$$\begin{aligned} I_{BC} &= Y_{BC}(v_B - v_C) \\ &= [g_{BC}\{v_{x(B)} - v_{x(C)}\} - b_{BC}\{v_{y(B)} - v_{y(C)}\}] \\ &\qquad + j[g_{BC}\{v_{y(B)} - v_{y(C)}\} + b_{BC}\{v_{x(B)} - v_{x(C)}\}] \end{aligned}$$

Let $-\sum\limits_{B} I_{BC} = I_{B1} = \{i_{x(B)1} + ji_{y(B)1}\}$ be the total current flowing into B from all the branches attached to it, so that,

$$\left.\begin{aligned} -i_{x(B)1} &= \sum_{B} [g_{BC}\{v_{x(B)} - v_{x(C)}\} - b_{BC}\{v_{y(B)} - v_{y(C)}\}] \\ \text{and} \qquad -i_{y(B)1} &= \sum_{B} [g_{BC}\{v_{y(B)} - v_{y(C)}\} + b_{BC}\{v_{x(B)} - v_{x(C)}\}] \end{aligned}\right\} \quad \dots \quad (3)$$

If $I_{B2} = i_{x(B)2} + ji_{y(B)2}$ stands for the current supplied to B from outside, then by Kirchoff's law—

$$\left.\begin{aligned} i_{x(B)} &= i_{x(B)1} + i_{x(B)2} = 0 \\ \text{and} \qquad i_{y(B)} &= i_{y(B)1} + i_{y(B)2} = 0 \end{aligned}\right\} \quad \dots \quad (4)$$

Supposing that the vector potential of D is unity and those of the points A, B, C to be zero, the currents passing from D along the branches DB and DC will be given by

$$\left.\begin{aligned} I_{DB} &= Y_{BD} = g_{BD} + jb_{BD} \\ \text{and} \qquad I_{DC} &= Y_{CD} = g_{CD} + jb_{CD} \end{aligned}\right\} \quad \dots \quad (5)$$

and no current will flow in any other branch of the circuit. In order that the assumed potential may be correct a current $-I_{D2} = I_{DB} + I_{DC} = i_{x(D)2} + ji_{y(D)2}$ will have to be supplied to D from outside. Under such a condition the currents I_{DB} and I_{DC} will leave the network at B and C respectively. But in fact no current passes to or leaves the network at B and C. Therefore it is required to superpose on the assumed potentials those which would result if currents $I_{B2} (= I_{DB})$ and $I_{C2} (= I_{DC})$ were supplied at B and C and allowed to leave the network at D and A the latter points being held at zero potential.

It is thus obtained initially

$$\begin{aligned} & i_{x(B)} = i_{x(B)2} = g_{BD}\,; \quad i_{x(C)} = i_{x(C)2} = g_{CD}\,; \\ & i_{y(B)} = i_{y(B)2} = b_{BD}\,; \quad i_{y(C)} = i_{y(C)2} = b_{CD}\,; \\ & i_{x(D)} = i_{x(D)2} = -(g_{BD} + g_{CD})\,; \\ & i_{y(D)} = i_{y(D)2} = -(b_{BD} + b_{CD}); \\ \text{and} \quad & i_{x(A)} = i_{y(A)} = 0. \end{aligned}$$

For liquidating these initial values (viz. $i_{x(B)}$, $i_{y(B)}$, $i_{x(C)}$, and $i_{y(C)}$) by imposing suitable vector potentials at B and C only, a table of standard operations can be prepared by the use of the following :

$$\left.\begin{aligned} \frac{\delta i_{x(C)}}{\delta v_{x(B)}} &= g_{BC} = \frac{\partial i_{y(C)}}{\partial v_{y(B)}}\ ; \\ -\frac{\partial i_{x(C)}}{\partial v_{y(B)}} &= b_{BC} = \frac{\partial i_{y(C)}}{\partial v_{x(B)}}\ ; \\ \text{and}\quad \frac{\partial i_{x(B)}}{\partial v_{x(B)}} &= \frac{\partial i_{y(B)}}{\partial v_{y(B)}} = -\sum_{B} (g_{BC})\ ; \\ -\frac{\partial i_{x(B)}}{\partial v_{y(B)}} &= \frac{\partial i_{y(B)}}{\partial v_{x(B)}} = -\sum_{B} (b_{BC})\ ; \end{aligned}\right\} \quad \ldots \quad (6)$$

which are easily obtained from relations (3).

On liquidation of the residuals (i.e., the initial values of $i_{x(B)}$, $i_{y(B)}$, $i_{x(C)}$ and $i_{y(C)}$) the vector values of the currents at A and D, and also of the potentials at B and C will be obtained These values will naturally correspond to the case when unit potential difference is applied between the terminals D and A.

The procedure involved in and the information available from such a method of solution will be apparent from the illustration given below :

ILLUSTRATION

For illustrating the method the following example which is worked out by Starr (1953), is considered.

Example. A three-phase transmission line 100 miles long has the following constants :

Resistance per mile = 0.25Ω; Reactance per mile = 0.8Ω; Susceptance per mile = 14×10^{-6} mho; Receiving end voltage = 66kv. It is required to determine (using nominal-T method),

(i) the sending end voltage and (ii) the sending end current when the line is delivering 15000 kw at 0.8 power factor lagging. The vector admittances of the different branches of the network (Fig. 2b), as calculated for the problem under consideration, are :

$$Y_{AB} = 10^{-3}(7.1-j22.79),$$

$$Y_{BC} = 10^{-3}(7.1-j22.79),$$

$$Y_{CD} = 10^{-3}(3.446-j2.58),$$

$$Y_{BD} = 10^{-3}(0+j1.4).$$

First of all it may be assumed for convenience that the potential at D is 1 kv (one kilovolt), and the potentials at A, B, and C are all zero. Then the currents in the branches DC and DB will be

$$I_{DC} = 3.446 - j2.58,$$

and
$$I_{DB} = 0 + j1.4.$$

Therefore, the current $-I_{D_2}$ to be fed from outside will be given by :

$$-I_{D_2} = 3.446 - j1.18.$$

Thus it is obtained initially,

$$i_{x(B)} = 0\ ;\ i_{x(C)} = 3.446;\quad i_{x(D)} = -3.446;$$

$$i_{y(B)} = 1.4;\quad i_{y(C)} = -2.58;\quad i_{y(D)} = 1.18;$$

and
$$i_{x(A)} = i_{y(A)} = 0.$$

Using the relations (6) and keeping in mind that all the 'g' and 'b' values mentioned there are to be multiplied by 10^3 (since here one kilo volt is considered in place of one volt mentioned previously) the unit operation table (Table I) is readily obtained for liquidating the initial values.

TABLE I

Unit operation table

	$\delta i_{x(A)}$	$\delta i_{x(B)}$	$\delta i_{x(C)}$	$\delta i_{x(D)}$	$\delta i_{y(A)}$	$\delta i_{y(B)}$	$\Delta i_{y(C)}$	$\delta i_{y(D)}$
$v_x(B)=1$	7.1	−14.2	7.1	0	−22.79	44.18	−22.79	1.4
$v_x(C)=1$	0	7.1	−10.546	3.446	0	−22.79	25.37	−2.58
$v_y(B)=1$	22.79	−44.18	22.79	−1.4	7.1	−14.2	7.1	0
$v_y(C)=1$	0	22.79	−25.37	2.58	0	7.1	−10.546	3.446

With the help of the procedure suggested previously by Basu (1958), the final operation table (Table II) is prepared which enables complete liquidation of the initial values in three steps only. It may, however, be mentioned that this procedure is not an essential one for the method under consideration. The relaxation table is then worked out as shown in Table III.

Thus when v_{DA} (i.e., sending end voltage) is 1 kv,

(i) the receiving end voltage,

$$v_{DC} = v_D - v_C = (0.788 - j0.143) \times 10^3 \text{ volts} = 0.8/\angle -10°18' \text{ kv}.$$

and

(ii) the sending end current,

$$I_A = I_D = 2.458 - j1.25$$
$$= 2.821/\angle -26°57' \text{ amps}.$$

TABLE II

Final operation table

$\delta v_{x(B)}$	$\delta v_{x(C)}$	$\delta v_{y(B)}$	$\delta v_{y(C)}$	$\delta i_{x(A)}$	$\delta i_{x(B)}$	$\delta i_{x(C)}$	$\delta i_{x(D)}$	$\delta i_{y(A)}$	$\delta i_{y(B)}$	$\delta i_{y(C)}$	$\delta i_{y(D)}$
.1	.2	0	0	.71	0	−1.4	.689	−2.279	−.14	2.795	−.376
−.188	−.377	.1	.194	.941	0	0	−.938	5.004	.22	−6.6	1.376
.275	.227	−.114	−.11	−.641	0	0	.636	−7.068	7.689	0	−.62

TABLE III

Relaxation table

$\delta v_{x(B)}$	$\delta v_{x(C)}$	$\delta v_{y(B)}$	$\delta v_{y(C)}$	$i_{x(A)}$	$i_{x(B)}$	$i_{x(C)}$	$i_{x(D)}$	$i_{y(A)}$	$i_{y(B)}$	$i_{y(C)}$	$i_{y(D)}$
Initial values				0	0	3.446	−3.446	0	1.4	−2.58	1.18
.246	.492	0	0	1.747	0	0	−1.751	−5.606	1.055	4.296	.255
−.122	−.245	.065	.126	2.359	0	0	−2.361	−2.353	1.198	0	1.149
−.043	−.035	.018	.017	2.458	0	0	−2.46	−1.257	0	0	1.242
.081	.212	.083	.143	2.458	0	0	−2.46	−1.257	0	0	1.242

Therefore for a receiving end voltage of $66/\sqrt{3}$ (between phase and neutral) the following quantities will be obtained,

Sending end current , $I_A = 134.4$ amps. [132.2 amps];

Sending end voltage, $v_{AD} = 82.5$ kv[82.3 kv];

and the sending end power factor $= \cos 26°57'$ lagging [$\cos 26°58'$].

The corresponding values of I_A, v_{AD} and the power factor as obtained by normal method of calculation (Starr, 1953) are given side by side within the brackets [ ], and it appears that they agree fairly well with each other.

DISCUSSION

The utility of the method described in this paper is appreciated when one considers that a relaxational solution as explained, readily yields a number of informations at a time viz., (i) vector currents at the sending and receiving ends, (ii) phase angle between the sending end and receiving end voltages together with the ratio between the two voltages, the ratio being constant for a given load-impedance, etc. Moreover the fact that the number of initial values to be liquidated in such problems will never exceed four, always brings forth a limitation in the labour associated with the process of liquidation. It may, however, be borne in mind that this method always requires a prior knowledge of the load admittance or the load impedance (which may be given directly or may be obtained from the given data). But as a rule the value of the receiving end impedance is not known unless the load, its power factor and the receiving end voltage are known. Under such conditions Case (ii), as mentioned earlier in the introduction, becomes identical with Case (i).

So far as the application of relaxation technique is concerned, the solution of long transmission lines by equivalent-T method is in no way different from the nominal-T method of solution of medium transmission lines. In the former case the branch impedances (i.e., Z_{AB}, Z_{BC}, and Z_{BD} of Fig. 1) only will have some changed values which can be calculated by using proper expressions given in standard text books.

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